

**DEPARTMENT OF COMPTER SCIENCE**

**UNIVERSITY OF KARACHI**

**OPERATION RESEARCH**

**PROJECT REPORT**

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**Project Report**

**Objective:**

To maximize the profit of XYZ paper mill gained by making notebook paper and newsprint.

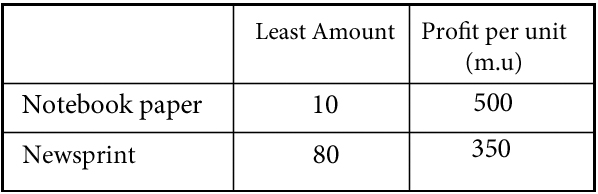
**Problem Definition:**

The XYZ paper mill wants to maximize its profit on each unit of making notebook paper and newsprint with a limited production of both.

**Analysis of the Problem:**

* The mill can product at most 200 units of paper a day for both notebook and newsprint.
* At least 10 units of notebook paper are required daily by customers.
* At least 80 units of newsprint are required daily by customers.
* The profit on a unit of notebook paper is 500 monetary units.
* The profit on a unit of newsprint is 350 monetary units.

By this Analysis, we can draw graphical structure as,



**Theory:**

To solve the described problem, We use Simplex Method of Linear Programming Approach.

**Linear Programming**

The linear programming model can be defined as the problem of minimizing and maximizing a linear function subject to linear constraints. The constraints may be equalities or inequalities.

**TERMINOLOGIES :-**

**Standard Constraint**

A constraint is said to be standard constraint when all variables are less than equal to total production per day (in this scenario).

**Standard Decision Variable**

Decision Variable is said to be Standard Decision Variable when all the Restriction/Decision Variables are greater than equal to Zero.

**Simplex Method**

A standard method of maximizing a linear function of several variables under several constraints on other linear functions is called **simplex method**. It provides us with a systematic way of examining the vertices of the feasible region to determine the optimal value of the objective function. The problem must be in the standard form in order to use of Simplex Method.

**Standard Form**

If all the constraints are in the form of equations and all the variables are non-negative then we can say that the linear program is in standard form.

**Objective Function**

The function which is either being minimizes or maximize. We often refer it as **Z**.

**Basic Solution**

A basic solution of a linear programming problem in standard form is a solution of the constraint equations in which at most m variables are nonzero–

**Basic Variables**

The variables that are nonzero are called basic variables.

**Non-Basic Variables**

Those variables whose values are zero are called non-basic variables.

**Slack/Surplus Variables**

It is a variable added or subtracted from the left hand side of less than equal to type constraint to convert it into equal sign. In general, Slack/Surplus Variable represent left or unused or overfilled capacity.

**Artificial Variables**

It is non-negative variable introduced to facilitate the computation of an initial basic feasible solution.

**Basic Feasible Solution**

A basic solution for which all variables are nonnegative is called a basic feasible solution. i.e Z function have all positive variables.

**Initial Basic Feasible Solution**

The first basic feasible solution which we get using conical form is called Initial Basic Feasible Solution.

**Solution:**

We can determine the maximam profit by using simplex method.

For solving with with simplex method we have to know its steps first.

**Algorithm of Simplex Method**

1. **Conversion to Conical From**
2. **Conversion to initial simplex method**
3. **Reporting the initial basic feasible solution.**
4. **Optimality Check**
5. **Improvement in the Current Basic Feasible Solution**
6. **Repeat step 4 and 5 continuously until objective function (Z) has all positive variables.**
7. **End.**

**IMPLEMENTATION:**

**Limitations/Constraints/Restrictions**

* **Production of paper 200 units Limited.**
* **Least amount of notebook paper production ( 10 units)**
* **Least amount of newsprint production ( 80 units).**

**Decision Variables:**

The profit will increase or decrease when we change the number of production of notebook paper or newsprint. So

Let

X1 = Notebook paper

X2 = Newsprints.

**Objective Function in terms of Decision Variable:**

**Z = 500 X1 + 350 X2**

Where,

Z = Maximum Profit.

**Constraints in Terms of Decision Variables:**

1. **X1 + X2 <= 200**

-maximum production constraint for both notebook paper and newsprint.

1. **X1 >= 10**

-notebook paper daily requirement constraint.

1. **X2 >= 80**

- newsprint daily requirement constraint.

**Restrictions in Terms of Decision Variables:**

1. **X1 >= 0**
2. **X2 >= 0**

**Linear Programming Model**

|  |
| --- |
| **To Max Z = 500 X1 + 350 X2**  **Subject to,**  **X1 + X2 <= 200**  **X1 >= 10**  **X2 >= 80**  **And**  **X1 >= 0**  **X2 >= 0** |

**Conversion to Conical Form**

**LP Model Conical Form**

**X1 + X2 <= 200 => X1 + X2 + S1 = 200**

**X1 >= 10 => X1 - S2 + V1 = 10**

**X2 >= 80 => X2 – S3 + V2 = 80**

Where,

S1,S2, S3 are **Slack/Surplus Variables**

V1,V2 are **Artificial Variables**.

**Minimize W = V1 + V2**

**Maximize W’ = - V1 - V2**

**PHASE I**

**Initial Simplex Table**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V1** | **V2** | **Z** | **W’** | **RHS** |
| **S1** | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 200 |
| **V1** | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 80 |
| **Z** | -500 | -350 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| **W’** | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |

V1 and V2 are converted into exact Basic Variable by applying row operations,

R5 = R5 – R2 ,

R5 = R5 – R3

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V1** | **V2** | **Z** | **W’** | **RHS** |
| **S1** | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 200 |
| **V1** | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 80 |
| **Z** | -500 | -350 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| **W’** | -1 | -1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | -90 |

For Pivot Element:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V1** | **V2** | **Z** | **W’** | **RHS** | **Ratio** |
| **S1** | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 200 | 200/1 |
| **V1** | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 10 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 80 | 80 |
| **Z** | -500 | -350 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | - |
| **W’** | -1 | -1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | -90 | - |

Selecting Pivot Column by choosing the highest negative value in Z.

Selecting Pivot Row by choosing the Least Ratio.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V1** | **V2** | **Z** | **W’** | **RHS** | **Ratio** |
| **S1** | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 200 | 200 |
| **V1** | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 10 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 80 | - |
| **Z** | -500 | -350 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | - |
| **W’** | -1 | -1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | -90 | - |

So.

Pivot Element = 1.

Applying Row operations,

R1 = R1 – R2

R4 = R4 – 500 R2

R5 = R5 + R2

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V1** | **V2** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 1 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 190 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 80 |
| **Z** | 0 | -350 | 0 | 500 | 0 | 500 | 0 | 1 | 0 | 5000 |
| **W’** | 0 | -1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | -80 |

Removing Artificial Variable column V1.

And Selecting Next Pivot Column by choosing the Only negative value in Z.

Selecting Pivot Row by choosing the Least Ratio.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V2** | **Z** | **W’** | **RHS** | **Ratio** |
| **S1** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 190 | 190 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 10 | - |
| **V2** | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | 80 | 80 |
| **Z** | 0 | -350 | 0 | 500 | 0 | 0 | 1 | 0 | 5000 | - |
| **W’** | 0 | -1 | 0 | 1 | 1 | 0 | 0 | 1 | -80 | - |

Pivot Element = 1.

Applying row operations,

R1 = R1 – R3

F4 = F4 + 350 R3

R5 = R5 + R3

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V2** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 0 | 1 | 1 | 1 | -1 | 0 | 0 | 110 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 10 |
| **V2** | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | 80 |
| **Z** | 0 | 0 | 0 | 500 | -350 | 350 | 1 | 0 | 33000 |
| **W’** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **V2** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 0 | 1 | 1 | 1 | -1 | 0 | 0 | 110 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 10 |
| **X2** | 0 | 1 | 0 | 0 | -1 | 1 | 0 | 0 | 80 |
| **Z** | 0 | 0 | 0 | 500 | -350 | 350 | 1 | 0 | 33000 |
| **W’** | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

V2 and W’ row will be removed.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 110 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 10 |
| **X2** | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 80 |
| **Z** | 0 | 0 | 0 | 500 | -350 | 1 | 0 | 33000 |

Artificial Variables are removed but Z row still contain one negative Value. So the solution is not Optimal.

**PHASE II**

Pivot Column is the one with the negative value in the Z row.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 110 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 10 |
| **X2** | 0 | 1 | 0 | 0 | -1 | 0 | 0 | 80 |
| **Z** | 0 | 0 | 0 | 500 | -350 | 1 | 0 | 33000 |

Applying row operations,

R3 = R3 + R1

R4 = R4 + 350 R1.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B V** | **X1** | **X2** | **S1** | **S2** | **S3** | **Z** | **W’** | **RHS** |
| **S1** | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 110 |
| **X1** | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 10 |
| **X2** | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 190 |
| **Z** | 0 | 0 | 350 | 850 | 0 | 1 | 0 | 71500 |

The Solution is **Optimal** Now, since the Z row contains all positive values.

**RESULT**

Z = 71500

At,

X1 = 10

X2 = 190

S1 = 0

S2 = 0

S3 = 110

**CONCLUSION**

The Maximum profit will be 71500 when the XYZ paper mill produce 10 unit of notebook papers and 190 units of newsprint.

**VERIFICATION**

**CONSTRAINT 1:**

**X1 + X2 <= 200**

**10 + 190 <= 200**

**200 <= 200**

**TRUE**

**CONSTRAINT 2:**

**X1 >= 10**

**10 >= 10**

**TRUE**

**CONSTRAINT 3:**

**X2 >= 80**

**190 >= 80**

**TRUE**

**Solution True for all Constraints hence Verified.**